Fermions, bosons, and locality in special relativity with two invariant scales

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(Dated: February 7, 2008)

We present a Master equation for description of fermions and bosons for special relativities with two invariant scales (c and λ_P). We introduce canonically-conjugate variables $\left(\chi^0, \boldsymbol{\chi}\right)$ to (ϵ, π) of Judes-Visser. Together, they bring in a formal element of linearity and locality in an otherwise non-linear and non-local theory. Special relativities with two invariant scales provide *all* corrections, say, to the standard model of the high energy physics, in terms of *one* fundamental constant, λ_P. It is emphasized that spacetime of special relativities with two invariant scales carries an intrinsic quantum-gravitational character.

PACS numbers: 03.30.+p, 04.50.+h, 04.60-m

Introduction.— There is a growing theoretical evidence that gravitational and quantum frameworks carry some elements of incompatibilities. The question is how deep are the indicated changes, and what precise form they may take. One hint comes from the observation that incorporating gravitational effects in quantum measurement of spacetime events leads to a Planck-scale saturation. In the framework of Kempf, Mangano, Mann, and one of us [1, 2], the gravitationally-induced modification to the de Broglie (dB) wave-particle duality takes the form [2]

$$\lambda_{dB} = \frac{h}{p} \quad \xrightarrow{grav.} \quad \lambda = \frac{\overline{\lambda}_P}{\tan^{-1}(\overline{\lambda}_P/\lambda_{dB})}, \tag{1}$$

where $\overline{\lambda}_P$ is the Planck circumference ($= 2\pi\lambda_P$), with $\lambda_P = \sqrt{\hbar G/c^3}$. The λ reduces to λ_{dB} for the low energy regime, and saturates to $4\lambda_P$ in the Planck realm. In this way the Planck scale is not merely a dimensional parameter but has been brought in relation to a universal saturation of gravitationally-modified de Broglie wavelengths.

This is a very welcome situation for theories of quantum gravity where for a long time a paradoxical situation had existed [3, 4]. Each inertial observer could measure in his frame the fundamental universal constants, $\hbar, c, G$, and obtain from them a universal fundamental constant, λ_P. And yet this very $\lambda_P -$ being a length scale $-$ is subject to special-relativistic length contraction which paradoxically makes it loose its universal character.

The indicated saturation then not only resolves this paradoxical situation but also suggests that special relativity must suffer a modification. This modification must be endowed with the property that it carries two invariant scales; one the usual c, and the second λ_P.

Amelino-Camelia, followed by Magueijo and Smolin, and Judes and Visser [3, 5, 6], have provided first steps towards development of a special relativity with two invariant scales (`SR2`); while Lukierski, Nowicki, and Kowalski-Glikman [4, 7] have brought to attention the underlying quantum/Hopf-group structure [8] of such theories.[1] The necessity for a SR2 as argued in Refs. [3, 4] is similar to ours, while motivation of Ref. [5] is contained in certain anomalies in astrophysical data [9, 10, 11, 12, 13]. Simplest of SR2 theories result from keeping the algebra of boost- and rotation- generators intact while modifying the boost parameter in a non-linear manner. Specifically, in the `SR2` of Amelino-Camelia the boost parameter, $\boldsymbol{\varphi}$, changes from the special relativistic form

$$\cosh\varphi = \frac{E}{m}\,, \quad \sinh\varphi = \frac{p}{m}\,, \quad \widehat{\boldsymbol{\varphi}} = \frac{\boldsymbol{p}}{p}\,. \tag{2}$$

to a new structure [6, 14]

$$\cosh\xi \;=\; \frac{1}{\mu}\left(\frac{e^{\lambda_P E} - \cosh\left(\lambda_P\, m\right)}{\lambda_P \cosh\left(\lambda_P\, m/2\right)}\right), \tag{3a}$$

$$\sinh\xi \;=\; \frac{1}{\mu}\left(\frac{p\, e^{\lambda_P E}}{\cosh\left(\lambda_P\, m/2\right)}\right), \quad \widehat{\boldsymbol{\xi}} = \frac{\boldsymbol{p}}{p}\,, \tag{3b}$$

[1] Instead of the term "doubly special relativity" coined in the work of Amelino-Camelia[5], we prefer to use the phrase "special relativity with two invariant scales." Without in any way questioning physics content of Amelino-Camelia's proposal, we take this non-semantic issue for the following reason. The special of "special relativity" refers to the circumstance that one restricts to a special class of inertial observers which move with a relative uniform velocity. The general of "general relativity" lifts this restrictions. The "special" of special relativity has nothing to do with one versus two invariants scales. It rather refers to the special class of inertial observers; a circumstance that remains unchanged in special relativity with two invariant scales. The theory of general relativity with two invariant scales would thus not be called "doubly general relativity."

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while for the SR2 of Magueijo and Smolin the change takes the form [3, 6]

$$\cosh\xi = \frac{1}{\mu}\left(\frac{E}{1-\lambda_P\,E}\right)\,, \tag{4a}$$

$$\sinh\xi = \frac{1}{\mu}\left(\frac{p}{1-\lambda_P\,E}\right)\,,\quad \widehat{\boldsymbol{\xi}} = \frac{\boldsymbol{p}}{p}\,. \tag{4b}$$

Here, μ is a Casimir invariant of SR2 (see Eq. (20) below) and is given by

$$\mu = \begin{cases} \frac{2}{\lambda_P}\,\sinh\left(\frac{\lambda_P\,m}{2}\right) & \text{for Ref. [5]'s SR2} \\ \frac{m}{1-\lambda_P m} & \text{for Ref. [3]'s SR2} \end{cases} \tag{5}$$

The notation is that of Ref. [6]; with the minor exceptions: λ, μ_0, m_0 there are λ_P, μ, m here. In what follows we shall *generically* represent boost parameter associated with special relativities with one, or two, invariant scales by $\boldsymbol{\xi}$. The former relativity shall be abbreviated as SR1 (to distinguish it from SR2).[2] Note that giving the explicit expressions for both the $\sinh\xi$ and $\cosh\xi$ in Eqs. (3a,3b) is necessary in order to fix the form of the energy-momentum dispersion relation through the identity: $\cosh^2\xi - \sinh^2\xi = 1$. Of course, one may have chosen to work in terms of one of the hyperbolic trigonometric functions *and* the dispersion relation, instead.

At this early stage it is not clear if there is a unique SR2, or, if the final choice will be eventually settled by observational data, or by some yet-unknown physical principle. Given this ambiguity, this *Letter* addresses itself to presenting a Master equations for fermionic and bosonic representations for generic SR2.

Master equation for spin-1/2: Dirac case.— Since the underlying spacetime symmetry generators remain unchanged much of the formal apparatus of the finite dimensional representation spaces associated with the Lorentz group remains intact. In particular, there still exist $(1/2,\,0)$ and $(0,\,1/2)$ spinors. But now they transform from the rest frame, to an inertial frame in which the particle has momentum, $\boldsymbol{p}$ as:

$$\phi_{(1/2,\,0)}(\boldsymbol{p}) = \exp\left(+\frac{\boldsymbol{\sigma}}{2}\cdot\boldsymbol{\xi}\right)\phi_{(1/2,0)}(\mathbf{0}) \tag{6a}$$

$$\phi_{(0,\,1/2)}(\boldsymbol{p}) = \exp\left(-\frac{\boldsymbol{\sigma}}{2}\cdot\boldsymbol{\xi}\right)\phi_{(1/2,0)}(\mathbf{0})\,. \tag{6b}$$

Since in this *Letter* we do not undertake a study of the behavior of these spinors under the parity operation, or examine the massless limit in detail, we do not identify the $(0,\,1/2)$ spinors as *left-handed* and the $(1/2,\,0)$ spinors as *right-handed.* Since the null momentum vector $\mathbf{0}$ is still isotropic, one may assume that (see p. 44 of Ref. [15] *and* Refs. [16, 17]):

$$\phi_{(0,1/2)}(\mathbf{0}) = \zeta\,\phi_{(1/2,0)}(\mathbf{0})\,, \tag{7}$$

where ζ is an undetermined phase factor. The analysis presented in Ref. [18] also convinces us that the validity of the identity (7) is independent of the "right-left" identification of the standard argument [15, 16, 17]. In general, the phase ζ encodes C, P, and T properties. The interplay of Eqs. (6a-6b) and (7) yields the Master equation for the $(1/2,\,0)\oplus(0,\,1/2)$ spinors,

$$\psi(\boldsymbol{p}) = \begin{pmatrix} \phi_{(1/2,\,0)}(\boldsymbol{p}) \\ \phi_{(0,\,1/2)}(\boldsymbol{p}) \end{pmatrix}\,, \tag{8}$$

to be

$$\begin{pmatrix} -\zeta & \exp(\boldsymbol{\sigma}\cdot\boldsymbol{\xi}) \\ \exp(-\boldsymbol{\sigma}\cdot\boldsymbol{\xi})\,\psi(\boldsymbol{p}) & -\zeta^{-1} \end{pmatrix}\psi(\boldsymbol{p}) = 0\,. \tag{9}$$

This is one of the central results of this *Letter.*

As a check, taking $\boldsymbol{\xi}$ to be $\boldsymbol{\varphi}$, and after some simple algebraic manipulations, the Master equation (9) reduces to:

$$\begin{pmatrix} -m\zeta & E\mathbb{1}_2 + \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ E\mathbb{1}_2 - \boldsymbol{\sigma}\cdot\boldsymbol{p} & -m\zeta^{-1} \end{pmatrix}\psi(\boldsymbol{p}) = 0\,, \tag{10}$$

where $\mathbb{1}_n$ stands for $n\times n$ identity matrix (and 0_n shall represent the corresponding null matrix) . With the given identification of the boost parameter we are in the realm of SR1. There, the operation of parity is well understood. Demanding parity covariance for Eq. (10), we obtain $\zeta = \pm 1$. Identifying

$$\begin{pmatrix} 0_2 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0_2 \end{pmatrix}\,,\quad \begin{pmatrix} 0_2 & -\boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0_2 \end{pmatrix}\,, \tag{11}$$

with the Weyl-representation γ^0, and γ^i, respectively, Eq. (10) reduces to the Dirac equation of SR1

$$(\gamma^\mu p_\mu \mp m)\,\psi(\boldsymbol{p}) = 0\,. \tag{12}$$

The linearity of the Dirac equation in, $p_\mu = (E, -\boldsymbol{p})$, is now clearly seen to be associated with two observations:

$\mathcal{O}_1$. That, $\boldsymbol{\sigma}^2 = \mathbb{1}_2$; and

$\mathcal{O}_2$. That in SR1, the hyperbolic functions – see Eq. (2) – associated with the boost parameter are linear in p_μ.

In SR2, observation $\mathcal{O}_1$ still holds. But, as Eqs. (3a - 3b) show, $\mathcal{O}_2$ is strongly violated. For this reason the Master equation (9) cannot be cast in a manifestly covariant form with a finite number of contracted Lorentz indices of SR2 as long as we mark spacetime events by x^μ of SR1.

The last inference is also a welcome result as it indicates a possible intrinsic non-locality in SR2s. Since in all SR2s the shortest spatial length scales that can be probed are bound from below by λ_P, the naively-expected $\delta^3(\boldsymbol{x}-\boldsymbol{x}')$ in the anticommutators of the form $\left\{\Psi_i(\boldsymbol{x},t)\,,\,\Psi_j^\dagger(\boldsymbol{x}',t)\right\}$ should be replaced by an highly,

[2] In this notation the Galilean relativity is denoted by SR0.

but not infinitely, peaked Gaussian-like functions with half-width of the order of λ_P.

The extension of the presented formalism for Majorana spinors is more subtle [19, 20, 21]. We hope to present it an extended version of this *Letter.*

Master equation for higher spins.— The above-outlined procedure applies to all, bosonic as well as fermionic, $(j,0)\oplus(0,j)$ representation spaces. It is not confined to $j=1/2$. A straightforward generalization of the $j=1/2$ analysis immediately yields the Master equation for an arbitrary-spin,

$$\left(\begin{array}{cc} -\zeta & \exp\left(2\boldsymbol{J}\cdot\boldsymbol{\xi}\right) \\ \exp\left(-2\boldsymbol{J}\cdot\boldsymbol{\xi}\right)\psi\left(\boldsymbol{p}\right) & -\zeta^{-1}\end{array}\right)\psi\left(\boldsymbol{p}\right)=0\,, \quad (13)$$

where

$$\psi\left(\boldsymbol{p}\right)=\left(\begin{array}{c}\phi_{(j,\,0)}\left(\boldsymbol{p}\right)\\ \phi_{(0,\,j)}\left(\boldsymbol{p}\right)\end{array}\right)\,. \quad (14)$$

Equation (13) contains the central result of the previous section as a special case. For studying the SR1 limit it is convenient to bifurcate the $(j,0)\oplus(0,j)$ space into two sectors by splitting the $2(2j+1)$ phases, ζ, into two sets: $(2j+1)$ phases ζ_+, and the other $(2j+1)$ phases ζ_-. Then, in particle's rest frame the $\psi(\boldsymbol{p})$ may be written as:

$$\psi_h(\boldsymbol{0})=\begin{cases}u_h(\boldsymbol{0}) & \text{when } \zeta=\zeta_+\\ v_h(\boldsymbol{0}) & \text{when } \zeta=\zeta_-\end{cases} \quad (15)$$

The explicit forms of $u_h(\boldsymbol{0})$ and $u_h(\boldsymbol{0})$ which we shall use (see Eq. (7)) are:

$$u_h(0)=\left(\begin{array}{c}\phi_h(\boldsymbol{0})\\ \zeta_+\,\phi_h(\boldsymbol{0})\end{array}\right),\; v_h(0)=\left(\begin{array}{c}\phi_h(\boldsymbol{0})\\ \zeta_-\,\phi_h(\boldsymbol{0})\end{array}\right), \quad (16)$$

where the $\phi_h(\boldsymbol{0})$ are defined as: $\boldsymbol{J}\cdot\widehat{\boldsymbol{p}}\,\phi_h(\boldsymbol{0})=h\,\phi_h(\boldsymbol{0})$, and $h=-j,-j+1,\ldots,+j$. In the parity covariant SR1 limit, we find $\zeta_+=+1$ while $\zeta_-=-1$.

As a check, for $j=1$, identification of $\boldsymbol{\xi}$ with $\boldsymbol{\varphi}$, and after implementing parity covariance, yields

$$\left(\gamma^{\mu\nu}p_\mu p_\nu\mp m^2\right)\psi(\boldsymbol{p})=0\,. \quad (17)$$

The $\gamma^{\mu\nu}$ are unitarily equivalent to those of Ref. [22], and thus we reproduce *bosonic matter fields* with $\{C,\,P\}=0$. A carefully taken massless limit then shows that the resulting equation is consistent with the free Maxwell equations of electrodynamics.

Since the $j=1/2$ and $j=1$ representation spaces of SR2 reduce to the Dirac and Maxwell descriptions, it is apparent, that the SR2 contains physics beyond the linear-group realizations of SR1. To the lowest order in λ_P, Eq. (9) yields

$$\left(\gamma^\mu p_\mu+\tilde{m}+\delta_1\,\lambda_P\right)\psi(\boldsymbol{p})=0\,, \quad (18a)$$

where

$$\tilde{m}\;=\;\left(\begin{array}{cc}-\zeta & 0_2\\ 0_2 & -\zeta^{-1}\end{array}\right)m \quad (18b)$$

and

$$\delta_1=\begin{cases}\gamma^0\left(\frac{E^2-m^2}{2}\right)+\gamma^i p_i\,E & \text{for Ref. [5]'s SR2}\\ \gamma^\mu p_\mu\,\left(E-m\right) & \text{for Ref. [3]'s SR2}\end{cases} \quad (18c)$$

Similarly, the presented Master equation can be used to obtain SR2's counterparts for Maxwell's electrodynamic. Unlike the Coleman-Glashow framework [23], the principle of special relativity with two invariant scales provides *all* corrections, say, to the standard model of the high energy physics, in terms of *one* – and *not forty six* – fundamental constant, λ_P.

Spin-1/2 and Spin-1 description in Judes-Visser Variables.— We now take the tentative position, that the ordinary energy-momentum p^μ is not the natural physical variable in SR2s. The Judes-Visser variables [6]: $\eta^\mu\equiv(\epsilon(E,p),\,\boldsymbol{\pi}(E,p))=(\eta^0,\boldsymbol{\eta})$ appear more suited to describe physics sensitive to Planck scale. The $\epsilon(E,p)$ and $\boldsymbol{\pi}(E,p)$ relate to the rapidity parameter $\boldsymbol{\xi}$ of SR2 in same functional form as do E and $\boldsymbol{p}$ to $\boldsymbol{\varphi}$ of SR1:

$$\cosh\left(\xi\right)=\frac{\epsilon(E,p)}{\mu}\,,\quad \sinh\left(\xi\right)=\frac{\pi(E,p)}{\mu}\,, \quad (19)$$

where

$$\mu^2=\left[\epsilon(E,p)\right]^2-\left[\boldsymbol{\pi}(E,p)\right]^2\,. \quad (20)$$

They provide the most economical and physically transparent formalism for representation space theory in SR2. For $j=1/2$ and $j=1$, Eq. (13) yields the *exact* SR2 equations for $\psi(\boldsymbol{\pi})$:

$$\left(\gamma^\mu\eta_\mu+\tilde{\mu}\right)\psi\left(\boldsymbol{\pi}\right)=0\,, \quad (21)$$

$$\left(\gamma^{\mu\nu}\eta_\mu\eta_\nu+\tilde{\mu}^2\right)\psi(\boldsymbol{\pi})=0\,, \quad (22)$$

where

$$\tilde{\mu}\;=\;\left(\begin{array}{cc}-\zeta^{-1} & 0_2\\ 0_2 & -\zeta\end{array}\right)\mu\,. \quad (23)$$

Concluding Remarks.— Our task in this *letter* was to provide a description of fermions and bosons at the level of representation space theory in SR2. However, we confined entirely to the representations of the type $(j,0)\oplus(0,j)$ – these types are important for matter fields, and to study gauge-field strength tensors. To study SR2's effect on the gauge fields and weak-field gravity the present *Letter's* formalism needs to be extended to (j,j) representation spaces. In view of Weinberg's earlier works [24] it is known that there is a deep connection between *local* quantum field theory, *SR1* (j,j) spaces [18], and the *equality* of the inertial and gravitational masses.

Therefore, the suggested study must answer `SR2`'s effect on the equivalence principle.

In quantum field theoretic framework, the special relativity's spacetime x^μ is canonically conjugate to p_μ, and appears in the field operators as:

$$\Psi(x) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{m}{p_0}\sum_{h=-j}^{+j}\Big[a_h(\boldsymbol{p})u_h(\boldsymbol{p})e^{-ip_\mu x^\mu} + b_h(\boldsymbol{p})v_h(\boldsymbol{p})e^{ip_\mu x^\mu}\Big]\,, \qquad (24)$$

where the particle-antiparticle spinors, $u_h(\boldsymbol{p})$ and $v_h(\boldsymbol{p})$ (generically represented by $\psi_h(\boldsymbol{p})$), are solutions of the Master equations (but with $\boldsymbol{\xi}\rightarrow\boldsymbol{\varphi}$) introduced above, and can be readily obtained from:

$$\psi_h(\boldsymbol{p}) = \begin{pmatrix} \exp(+\boldsymbol{J}\cdot\boldsymbol{\varphi}) & 0_{2j+1} \\ 0_{2j+1} & \exp(-\boldsymbol{J}\cdot\boldsymbol{\varphi}) \end{pmatrix}\psi_h(\boldsymbol{0})\,. \qquad (25)$$

Now, as our discussion on non-locality indicates x^μ of `SR1` is perhaps not the natural physical spacetime variable at the Planck scale. The spacetime at Planck scale, we suggest, is represented by new event vectors χ^μ (to be treated as "canonically conjugate" to Judes-Visser variable η_μ); and suggests the following definition for the field operators built upon the SR2's spinors:

$$\Psi(\chi) = \int \frac{d^3\boldsymbol{\eta}}{(2\pi)^3}\frac{\mu}{\eta_0}\sum_{h=-j}^{+j}\Big[a_h(\boldsymbol{\eta})u_h(\boldsymbol{\eta})e^{-i\eta_\mu \chi^\mu} + b_h(\boldsymbol{\eta})v_h(\boldsymbol{\eta})e^{i\eta_\mu \chi^\mu}\Big]\,, \qquad (26)$$

with

$$\psi_h(\boldsymbol{\eta}) = \begin{pmatrix} \exp(+\boldsymbol{J}\cdot\boldsymbol{\xi}) & 0_{2j+1} \\ 0_{2j+1} & \exp(-\boldsymbol{J}\cdot\boldsymbol{\xi}) \end{pmatrix}\psi_h(\boldsymbol{0})\,. \qquad (27)$$

Immediately, we verify that for spin-1/2 fermions in SR2

$$\left\{\Psi_i\left(\boldsymbol{\chi},\chi^0\right),\,\Psi_j^\dagger\left(\boldsymbol{\chi}',\chi^0\right)\right\} = \delta^3\left(\boldsymbol{\chi}-\boldsymbol{\chi}'\right)\,\delta_{ij}\,. \qquad (28)$$

What appears as non-locality in the space of events marked by x^μ now, in the space of events marked by χ^μ, exhibits itself as locality. This is a rather unexpected observation and it calls for a deeper understanding of the η_μ and χ^μ description of SR2. The Planck length is intrinsically built in the latter spacetime variables, and it may carry significant relevance for extending SR2 to the gravitational realm.

The evolution of special relativity in the sequence[3]

$$\texttt{SR0}\xrightarrow{c}\texttt{SR1}\xrightarrow{c,\lambda_P}\texttt{SR2} \qquad (29)$$

translates to giving spacetime, first, a *relativistic* and, then, a *quantum-gravitational* character. The work initiated here, and in Ref. [25], gives concrete shape to modifications that one may expect in the standard model of high-energy physics and theory of gravitation.

Acknowledgments.— One of us (DVA) thanks Giovanni Amelino-Camelia, Gaetano Lambiase, and their respective institutes in Rome and Salerno, for discussions and hospitality in May 2001. It is also our pleasure to record that part of this work was initiated during those visits, and that concurrently Arzano and Amelino-Camelia [25] have obtained some results similar to the ones reported here. We are also grateful to Giovanni for freely exchanging with us his thoughts on *Doubly Special Relativity* and for engaging in time-consuming extended discussions on the material presented here.

This work is supported by Consejo Nacional de Ciencia y Tecnología (CONACyT, Mexico).

[3] The symbols above the arrows indicate the invariants for the subsequent `SRn`.

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